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Exploring Teaching Witnesses' Feedback in the Teaching Experiments Phase of Design-based Research for Differential Calculus Instructional Lessons

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ABSTRACT

This study assesses the strengths and weaknesses of the designed Differential Calculus Instructional Lessons (DCIL) for teaching differential calculus from the perspective of teaching witnesses. Additionally, the study also focused on identifying suggested improvements from these teaching witnesses to refine DCIL. The research employed a design-based research approach, and this study focused only on outcomes from semi-structured interviews with teaching witnesses who participated in separate teaching experiment cycles and their evaluation of the content validity of DCIL on the feedback form. Findings revealed that the strengths of the instructional lessons encompassed lesson effectiveness, interactive teaching strategies, and effective technology use. The results also highlighted specific challenges hindering students' understanding of differentiation topics, including content overload and time constraints, difficulties in understanding differentiation concepts, and limited feedback and interaction on the Desmos platform. The teaching witnesses recommended two major enhancements: improving student engagement and interaction and enhancing the Desmos platform for more effective teaching practices. The study exemplifies collaborative design-based research where the researcher engages course coordinators as teaching witnesses throughout two teaching cycles and contributes insights for refining the designed DCIL in teaching and learning differential calculus.

Keywords: Differential calculus, design-based research, Differential Calculus Instructional Lessons (DCIL), Mathematics education, teaching experiment, teaching witnesses

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INTRODUCTION

Differential calculus, a fundamental branch of calculus distinct from integral calculus, is essential for understanding various mathematical concepts such as ordinary differential equations and multivariate calculus (Hamda et al., 2020; Kwon et al., 2015; Mcgee & Moore-Russo, 2015). Mastering this subject is crucial for students, particularly those in pre-university programs, as it forms the basis for university-level studies. However, students often perceive calculus as challenging, primarily due to difficulties in understanding and problem-solving (Hashemi et al., 2020; Jones & Watson, 2018; Wagner et al., 2017).

Recent research highlighted a significant gap in mathematics education, where instructional materials often prioritised procedural knowledge, which means the educators placed more emphasis on memorisation and repetitive practice than on helping students develop deeper conceptual understanding through problem-solving (Hamid et al., 2021). This focus on procedural skills restricted students' ability to think critically and apply mathematical concepts in diverse contexts. Studies by Makgakga and Makwakwa (2016), Othman et al. (2018), and Setiawan (2022) indicated that this overreliance on memorisation impeded effective learning and failed to promote a comprehensive understanding of mathematical principles. Additionally, Brijlall and Ndlazi (2019) and Wagner et al. (2017) found that an emphasis on symbolic manipulations and neglected graphical representations can hinder students' comprehension by not offering them multiple ways to visualise and understand mathematical concepts.

Educators were encouraged to integrate innovative teaching methods and technology into their practices to address these issues (Malaysian Qualifications Agency, 2019). The technology offered the potential for dynamic and interactive learning experiences that could bridge the gap between procedural knowledge and conceptual understanding, such as through visualisation and personalised learning. However, there was a notable gap in the effective integration of technology in teaching due to a lack of specific guidelines (Bedada & Machaba, 2022).

Design-based research has increasingly been utilised in educational research (The Design-Based Research Collective, 2003). It offers a framework for exploring innovative teaching methods and their impact on student learning (Molina et al., 2007). In this study, an instructional unit named Differential Calculus Instructional Lesson (DCIL) is designed, and a research team conducted design-based research in real classroom settings to test and revise this DCIL. The team included the researcher as the instructor who implemented the designed instructional lessons and the course coordinator as a teaching witness, primarily responsible for observing the lessons during the teaching experiment phase (Cobb et al., 2003). While many studies focus on technology integration or innovative teaching methods (Haryani & Hamidah, 2022; Koller et al., 2008; Shé, Fhloinn, & Bhaird, 2023), few studies explore how teaching witnesses contribute to both pedagogy and the iterative design of educational interventions in calculus education. This study addresses the need for more evidence on how real-time feedback from teaching witnesses can be integrated into an ongoing design-based process, filling a gap in understanding how continuous observation and feedback impact student outcomes.

In the context of design-based research (DBR), teaching witnesses play significant roles in improving instructional methods and student learning experiences. DBR is an iterative process that involves collaboration between educators, who act as teaching witnesses and researchers to design, implement, and refine the intervention. Teaching witnesses, on the other hand, serve as observers to reflect on these educational interventions (Norton & McCloskey, 2008). By offering firsthand insights, they help to ensure that the implementation of new strategies or interventions is both effective and reflective of diverse learning needs (Bungum & Sanne, 2021). It is believed that the combination of DBR's systematic refinement process and teaching witnesses' practical observations is able to create a dynamic framework for advancing calculus pedagogy and enabling educators to better address complex mathematical concepts while catering to varying student abilities and learning styles.

The primary objective of the study is to investigate feedback from the teaching witnesses on the designed Differential Calculus Instructional Lessons (DCIL). The study also identified suggested enhancements by teaching witnesses for continuous improvement and modification of DCIL and addresses the following research questions:

Based on the teaching witnesses' feedback:

RQ1: What are the notable strengths of these designed instructional lessons?

RQ2: What are the identified weaknesses of these instructional lessons?

RQ3: What suggested enhancements can be made to the lesson to optimise its effectiveness?

LITERATURE REVIEW

Calculus

Differential calculus, a fundamental aspect of mathematics education, presents significant challenges for students, as identified in several studies (Bakri et al., 2020; Fatimah & Yerizon, 2019; Makgakga & Makwakwa, 2016). These difficulties arise from factors such as the introduction of new concepts and inadequate prior knowledge in integrating pre-calculus ideas with calculus (Fitriani et al., 2023). Proficiency in this field requires a deep conceptual understanding and skilled problem-solving skills, which many students struggle to attain due to insufficient understanding of essential elements (Bibi et al., 2019).

Numerous investigations have delved into students' understanding of derivatives, revealing cognitive barriers in integrating pre-calculus concepts and grappling with calculus principles (Hitt & Dufour, 2021). Similarly, Meiliasari and colleagues

(2021) identified students frequently fail to summarise their responses, and this leads to confusion when prompted for conclusions as they struggle to understand the fundamental concept of differentiation. Research indicates that students often encounter difficulties in understanding fundamental derivative concepts across various contexts, such as numerical, physical, verbal, and graphical (Haghjoo & Reyhani, 2021).

The framework of "concept image" and "concept definition" by Tall and Vinner (1981) has significantly influenced the understanding of how students grasp advanced mathematical concepts, particularly in calculus. This framework reveals that students' mental representations often do not align with formal mathematical definitions, leading to errors or misconceptions (Makonye & Luneta, 2014). For instance, the discrepancy between the formal definition and a student's concept image can cause difficulties when learning concepts like differential calculus, limits, and continuity (Tall & Vinner, 1981).

A study by Ojo and Olanipekun (2023) further emphasised that students frequently rely on a mix of correct and incorrect concept images, which can result in both accurate and flawed reasoning in calculus. For example, a student may correctly perceive a derivative as a slope but might misunderstand its broader applications due to incomplete or incorrect concept images. These issues often arise from a procedural focus in mathematics education, where students prioritise memorising steps over understanding the underlying principles. Therefore, by understanding how concept image influences learning outcomes, educators can design targeted interventions to support students' understanding of calculus.

Few research suggested that the utilisation of visual reasoning skills and graphs in mathematics classrooms proved beneficial in helping students comprehend derivative concepts (Aspinwall et al., 1997; Borji et al., 2018; Hamid et al., 2021). These studies highlighted that incorporating technology into calculus instruction has shown promise in facilitating learning. Platforms like YouTube provide accessible resources for concept review (Lu, 2023), while graphic display software such as Desmos enables students to interact with visual representations and reinforce their conceptual understanding (Chechan et al., 2023; Chorney, 2021). Moreover, dynamic software like GeoGebra and SimCalc MathWorlds offer interactive experiences, fostering constructivist learning and promoting dialogue (Arango et al., 2015; Salinas et al., 2016). The literature strongly supports the benefits of incorporating technology in mathematics instruction, highlighting the importance of including course lecturers' opinions and feedback in the design process. Their input enhances teaching and learning and aids in refining instructional materials (Shahid et al., 2022).

Calculus in Design-based Research

Design-based Research (DBR) is an approach involving a systematic design process, and it has been employed to investigate student understanding of calculus (Hamda et al., 2020; Pramesti & Dewi, 2023). For example, Hamda et al. (2020) utilised a design research approach to introduce the concept of the limit of a sequence using popular characters from a children's television show. Similarly, Keene et al. (2014) implemented an inquiry-based multivariable calculus course to enhance students' argumentation skills. Interventions by Block and Mercorelli (2015) effectively improved students' arguments and understanding.

Studies examined various aspects of instructional design in calculus, such as teaching integration by parts (Mariano et al., 2021) and observing active learning engineering calculus classrooms (Shahid et al., 2022). Others proposed hypothetical learning trajectories to enhance students' argumentative skills and understanding of multivariable calculus (Hamda et al., 2020). These studies contributed valuable insights into diverse approaches and considerations for effective instructional strategies in teaching calculus. This subject requires careful planning and execution of lessons to enhance student understanding and engagement.

METHODS

The research methodology employed in this study is structured around design-based research (DBR), consisting of three phases: preparation and design, conducting a teaching experiment, and carrying out a retrospective analysis (Bakker & van Eerde, 2015; Steffe & Thompson, 2000). This approach involves an iterative cycle of designing, testing, evaluating, and refining the intervention in real-world contexts (Goff & Getenet, 2017; Koller et al., 2008). In each cycle of DBR, researchers collaborate closely with a teacher or lecturer, serving as a teaching witness, to identify problems, devise solutions, and refine interventions based on the evidence gathered (Kennedy-Clark, 2013). This paper focused specifically on evaluating the efficacy of Differential Calculus Instructional Lessons (DCIL) for teaching differential calculus. The evaluation was conducted only from the perspective of teaching witnesses, who observed the teaching experiment phases.

Unlike traditional experimental approaches, the teaching experiment methodology involved the researcher actively taking on the role of the instructor while closely collaborating with course lecturers who served as teaching witnesses (Molina et al., 2007). This collaboration enabled real-time reflection and refinement of instructional interventions, as they provided valuable insights into students' understanding and learning processes (Cobb et al., 2003; Stephan, 2015). These collaborative discussions between researchers and teaching witnesses were essential due to the contextual nature of DBR and the teaching witnesses' familiarity with their students' learning styles. Bungum and Sanne (2021) further highlighted teaching witnesses as active 'co-designers,' crucial for embedding DBR outcomes into teachers' practice, thereby enhancing learning experiences, fostering effective collaboration, and aligning interventions with actual educational needs.

The teaching experiment comprised two cycles, each consisting of four instructional lessons, following DBR principles (Bakker & van Eerde, 2015). The study was conducted with a group of 16 foundations in science students from a private university, selected through purposive sampling to ensure diversity in achievement levels (Smith & Strahan, 2004). Students interacted with instructional content using individual devices equipped with the Desmos platform, fostering interactive learning experiences tailored to their needs. Each lesson was conducted over two hours. The content of the instructional lessons focused on curve properties through the analysis of first and secondary derivatives, as well as the application of differential calculus concepts via graphical representations from the Desmos platform.

During each cycle, discussions were held with course coordinators, serving as teaching witnesses, to assess lesson effectiveness and identify areas for improvement. Teaching Witness 1, an experienced lecturer with 10 years of teaching experience, and Teaching Witness 2, a relatively new lecturer with about two years of experience at the selected institution, observed all teaching episodes within their respective cycles to gain firsthand insights into student learning and operational dynamics. Although Teaching Witness 2 has only two years of experience at the selected institution, she had an additional two years of pre-university teaching experience at another institution before joining the current one. According to Melnick and Meister (2008), individuals with 4 to 37 years of experience are considered experienced teachers, which qualifies Teaching Witness 2 as an experienced lecturer in teaching mathematics. Another reason for selecting these two lecturers as teaching witnesses for this study is that they were both mathematics course coordinators at the time of the study, allowing them to provide more constructive opinions and feedback on the lessons, given their in-depth knowledge of the course. In subsequent discussion, Teaching Witness 1 referred to the lecturer who observed during the first cycle, while Teaching Witness 2 observed during the second cycle.

This study is limited in scope, focusing solely on the feedback of two teaching witnesses regarding the designed DCIL. The analysis primarily focused on interviews conducted with the teaching witnesses, providing insights into the strengths and weaknesses of the instructional lessons and suggesting improvements to enhance teaching and learning. The semi-structured interviews were conducted with the teachers on the day following each lesson and after the completion of each cycle to gather specific feedback (Rashidi et al., 2014). Additionally, feedback forms were completed after each teaching episode to assess the content validity of the designed instructional lessons.

Data Analysis

Content validity of Differential Calculus Instructional Lessons (DCIL) will be assessed using a validation form, employing the Percentage Calculation Method (PCM) formulas as follows:

$$\text{Content Validity Level} = \frac{\text{Total score given by the expert}}{\text{Total score}} \times 100\% \quad (1)$$

The total score is determined by multiplying the number of items by the maximum score on the Likert scale. An instructional lesson is considered to have high content validity if the content validity score is 70% or above (Jamaludin, 2016).

The semi-structured interviews with two teaching witnesses were recorded and transcribed for analysis with ATLAS.ti 23 software, following Braun and Clarke's (2006) thematic analysis method. Thematic analysis involves becoming familiar with the data, generating initial codes, searching for themes, defining and naming those themes, and finally, reporting patterns (themes) within the data. The "member check" concept described by McKim (2023) and Thomas (2017) was applied, and transcripts were given to participants for feedback and corrections to ensure data accuracy and reliability, as recommended by Bogdan and Biklen (2007). This iterative process enhances the validation and trustworthiness of the collected qualitative data.

RESULTS

Measurement of Content Validity

Two teaching witnesses, each in one cycle, witnessed the classroom condition and gave feedback on the designed Differential Calculus Instructional Lessons (DCIL). These teaching witnesses then filled out a feedback form during each teaching episode, and they were interviewed by the researcher a day after the teaching episodes. In this session, their feedback on the teaching episodes and the witness feedback form will be detailed. The scores shown in Table 1 were the average scores from the four feedback forms provided by each teaching witness, using a scale ranging from 1 (denoting strongly disagree) to 10 (indicating strongly agree).

The achievement of content validation is calculated using the Percentage Calculation Method (PCM). The good content validation percentage was set at 70%. The overall performance for this designed DCIL has achieved the content validity of 87.34% with a coefficient value of 0.87, above 70% or 0.70. Based on the result, the contents in the DCIL are considered to be of good validity. Table 1 shows the content validity measurement for the designed DCIL.

Teaching Episodes Witness Interviews

The teachers were interviewed the day after each teaching episode to gather their feedback on the designed lesson. The overall feedback interview was also conducted after the whole cycle of DCIL implementation was done. The interview was held in the meeting room because the venue was quiet and the room was unoccupied. The teachers gave their responses to the interview questions based on their observations throughout the whole lesson in the classroom.

Table 1
Content validity measurement for the DCIL

Items	Teaching Witness 1	Teaching Witness 2
The content of this instructional lesson is appropriate for the student's educational level.	8.25	8.75
The content of this instructional lesson can be successfully implemented.	9	8.75
The content of this instructional lesson is suitable for the allocated time frame.	8	8.5
The content of this instructional lesson aligns with the syllabus for the Differentiation chapter in the Mathematics course.	9	9
The content of this instructional lesson meets the research objectives, specifically assessing students' mathematical understanding based on APOS theory.	8.5	9.25
The content of this instructional lesson incorporates Desmos instructional activities related to the topic of Differentiation.	8	8.5
The content of this instructional lesson can enhance students' mathematical understanding.	8.5	9.25
The content of this instructional lesson uses the correct terminology outlined in the Mathematics course syllabus for the pre-university program.	8.75	9.75
Total	68	71.75
Content Validity Achievement (100%)	87.34%	
Content Validity Coefficients (1.00)	0.87	

Each transcript was carefully analysed, and initial codes were assigned to significant statements made by the teaching witnesses. These codes were then grouped into categories, which were subsequently aligned with the research questions as predetermined themes. The outcomes of the interview were categorised into three primary themes to gain insights into the strengths and weaknesses of the designed DCIL lessons, as well as to propose improvements in lesson feedback and interaction.

Strengths of DCIL Lesson

Figure 1 shows the analysis of the interview transcripts from ATLAS.ti, which revealed three categories of the strengths of the designed DCIL lessons: Lesson Effectiveness and Achievement, Interactive Teaching Strategies and Effective Technology Use.

For the lesson effectiveness and achievement, the feedback gathered from the teaching witness's interview transcripts consistently highlights positive aspects regarding the overall structure, content suitability, and the achievement of learning outcomes. The lessons, particularly those in Lesson 2, were commended for their effectiveness in meeting the objectives outlined in the syllabus. For example, Teaching Witness 1 explained the suitability of the lesson's content as follows:

> *Researcher: Today, we are going to reflect on lesson 2.*
>
> *Teaching Witness 1: Okay, for this lesson, in section A, I found that the*

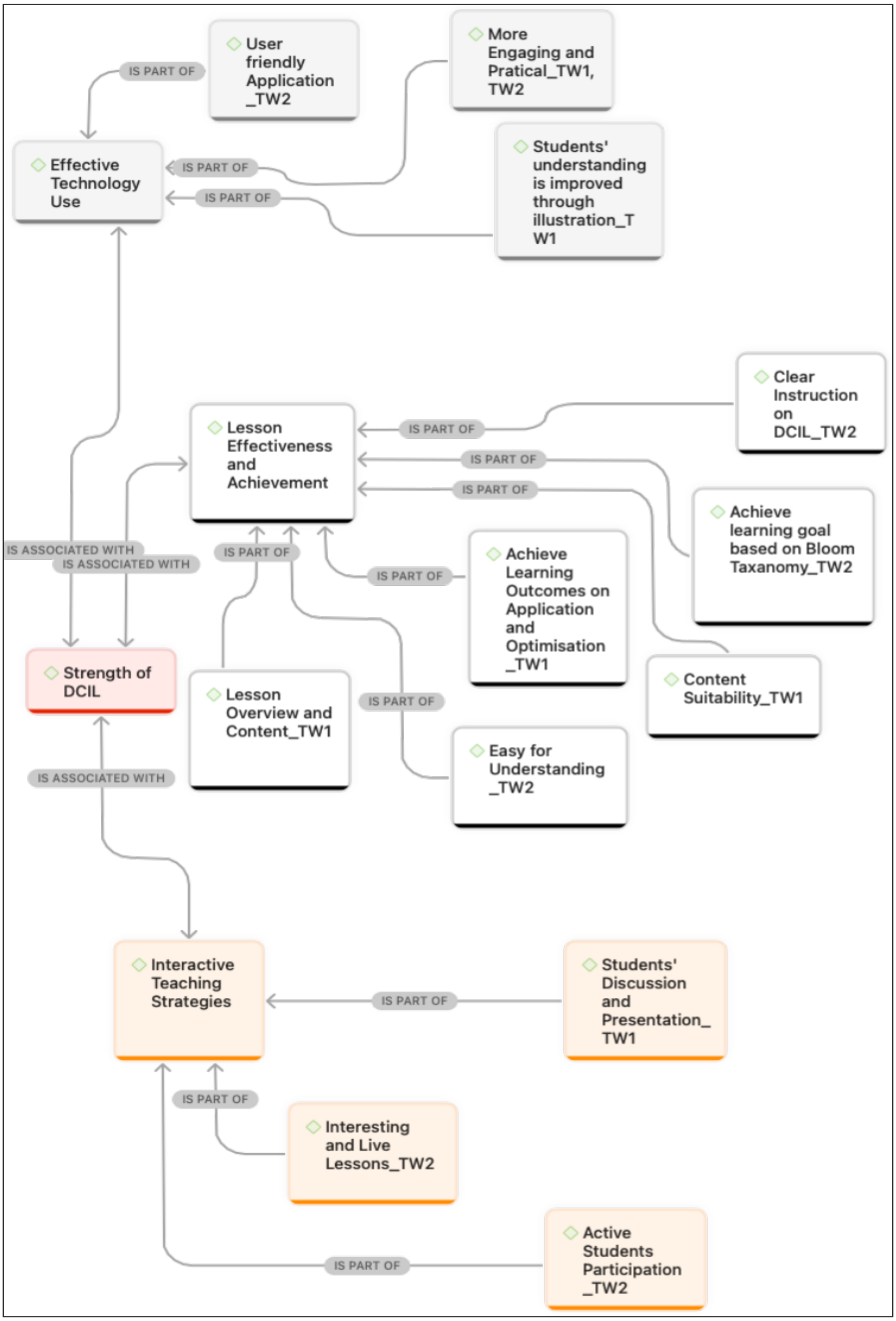

Figure 1. The strengths of the DCIL

> *content is suitable* (Interview L2_TW1: 3-4).

> *Teaching Witness 1: We are able to achieve the lesson as well as the learning outcome, especially the differentiation offered in the mathematics syllabus in the foundation* (Interview L2_TW1: 6).

Teaching Witness 2 expressed that the lessons were delivered exceptionally well, with an emphasis on clarity and ease of understanding.

> *Teaching Witness 2: Your syllabus is delivered very well and is easy to understand.* (Interview L2_TW2: 21)

Furthermore, Teaching Witness 2 emphasised the effectiveness of the DCIL in attaining the lesson objectives, specifically within the framework of Bloom's Taxonomy, with a notable focus on its application in the field of differentiation topics.

> *Teaching Witness 2: This platform is also suitable for achieving our goals in Bloom's Taxonomy, particularly in the application of mathematics. The students were able to apply derivatives in practical contexts* (Interview Overall_TW2: 10).

According to teaching witness 1, the success in quadratic and cubic graph matching highlighted a specific achievement within the DCIL lessons. The lesson content, focusing on optimisation techniques, required students to apply their knowledge in employing optimisation methods to determine maximum or minimum values. Teaching Witness 1 positively highlighted that those students excelled in matching quadratic and cubic graphs, further emphasising the DCIL's effectiveness in delivering content with a practical and application-oriented approach.

> *Teaching Witness 1: ... they were able to match the cubic graph with the quadratic derivative* (Interview L1_TW1: 66).

Teaching Witness 2 also acknowledged the clarity and helpfulness of the content. Specifically, the inclusion of concise notes with clear instructions was recognised as a significant contributor to the positive learning experience.

> *Teaching Witness 2: ... The content is quite interesting because, inside the instructional unit, short notes were provided with perfect instructions* (Interview L3_TW2: 35)

The second strength of the designed DCIL lessons lies in their demonstration of the effective use of interactive teaching strategies. The integration of discussions, presentations, and student participation was specifically identified as a strength of the designed DCIL lessons. The use of Desmos allowed students to interactively draw, sketch, and manipulate graphs, adding a layer of interest and liveliness to the lessons. Teaching Witness 1 acknowledged the valuable discussions that occurred at the end of the session. These discussions not only facilitated student understanding but also actively engaged the students. Furthermore, the presentation by one group became a vehicle for students to showcase their understanding of the materials, leading to increased confidence in presenting concepts before their peers.

Teaching Witness 1: Furthermore, the instructor provided some time for the students to discuss in a group, and then the students could present their ideas in front (Interview L4_TW1: 6).

Teaching Witness 1: At the end of the session, although we had limited time, at least one group was able to present their results.

Researcher: Yes.

Teaching Witness 1: I believe that through this session, the students would have understood more and could also build their confidence (Interview L3_TW1: 15-17).

Teaching Witness 2 also highlighted similar observations to those noted by Teaching Witness 1. The lessons actively promoted student involvement, allowing students to draw and sketch graphs during the session and submit their answers. Learning from their mistakes was facilitated as the instructor showcased solutions provided by all students.

Teaching Witness 2: ... Overall, it fulfils their learning needs by promoting engagement rather than just passive listening (Interview Overall_TW2: 6)

Teaching Witness 2: ...This feature encourages them to submit answers, learn from their mistakes, and enhance critical thinking (Interview Overall_ TW2: 8).

Another prominent feature of the designed DCIL lesson was the belief in the effectiveness of technology integration. According to Teaching Witness 1, using technology, including computers and the Desmos platform, proved to be an effective strategy in making the learning experience more engaging and practical compared to traditional methods like graph paper and pencils.

Teaching Witness 1: And the thing is a live thing because they were able to draw, to sketch, and they were also able to move the mouse here and there to see how the graph looks like, and so on.

Teaching Witness 1: Rather than just using graph paper or pencils to draw or imagine how the graph looks (Interview L2_TW1: 10 & 14).

Teaching Witness 1 noted an improvement in student understanding through the use of illustrations from Desmos. This integration significantly enhanced student comprehension and mastery of the subject matter. Because the method supplemented traditional instruction with diagrams and visual aids, students gained the ability to perform calculations and developed a deeper conceptualisation and visualisation of the underlying concepts.

Teaching Witness 1: I can see that it was very interactive because you included pictures inside the Desmos app. It had step-by-step pictures.

Researcher: Yes.

Teaching Witness 1: as well as diagrams. So, I believe the students would be more understanding because they could not only calculate but also imagine what was happening at the same time with the diagrams provided. (Interview L3_TW1: 9-11)

Teaching Witness 1: I believe that the diagrams and illustrations help the students understand the application of optimisation easily (Interview L4_TW1: 6).

Using illustrations enables students to understand the practical applications of optimisation with greater ease. Teaching Witness 2 repeated similar thoughts, noting that students could identify specific values of the function and trace the position of the derivative function using the Desmos platform. This resulted in enhanced problem-solving strategies, encompassing tasks such as maximising or minimising, understanding constants, and effectively drawing and labelling diagrams.

Teaching Witness 2: I find it useful to identify certain values of the function.

Researcher: Okay.

Teaching Witness 2: That's one of the first advantages. Another one is that students can explain the process used to arrive at the solution, and they're able to trace the position of the derivative function.

Researcher: So, does this platform help them identify the derivative function?

Teaching Witness 2: Ah, yes (Interview L2_TW2: 5-9).

Teaching Witness 2: Yes, which means they can identify what to maximise or minimise, understand the constants, and know how to draw and label diagrams.(Interview L3_TW2: 11)

Weaknesses of DCIL Lesson

During the interview, both teaching witnesses highlighted several weaknesses in the designed Differential Calculus Instructional Lesson (DCIL) based on their assessment of the four teaching episodes. Figure 2 shows an analysis of the interview script that revealed three primary sub-themes encompassing these weaknesses: Content Overload and Time Constraints, Students' Difficulties in Understanding Differentiation Concepts, and Limited Feedback and Interaction on the Desmos platform.

According to Teaching Witness 1, the two-hour lesson might not provide sufficient time for students to fully understand and process the content, especially when matching graphs involving linear, quadratic, and cubic functions with their derivatives in Lesson 1.

Teaching Witness 1: But I notice that, perhaps, it is too much for the students

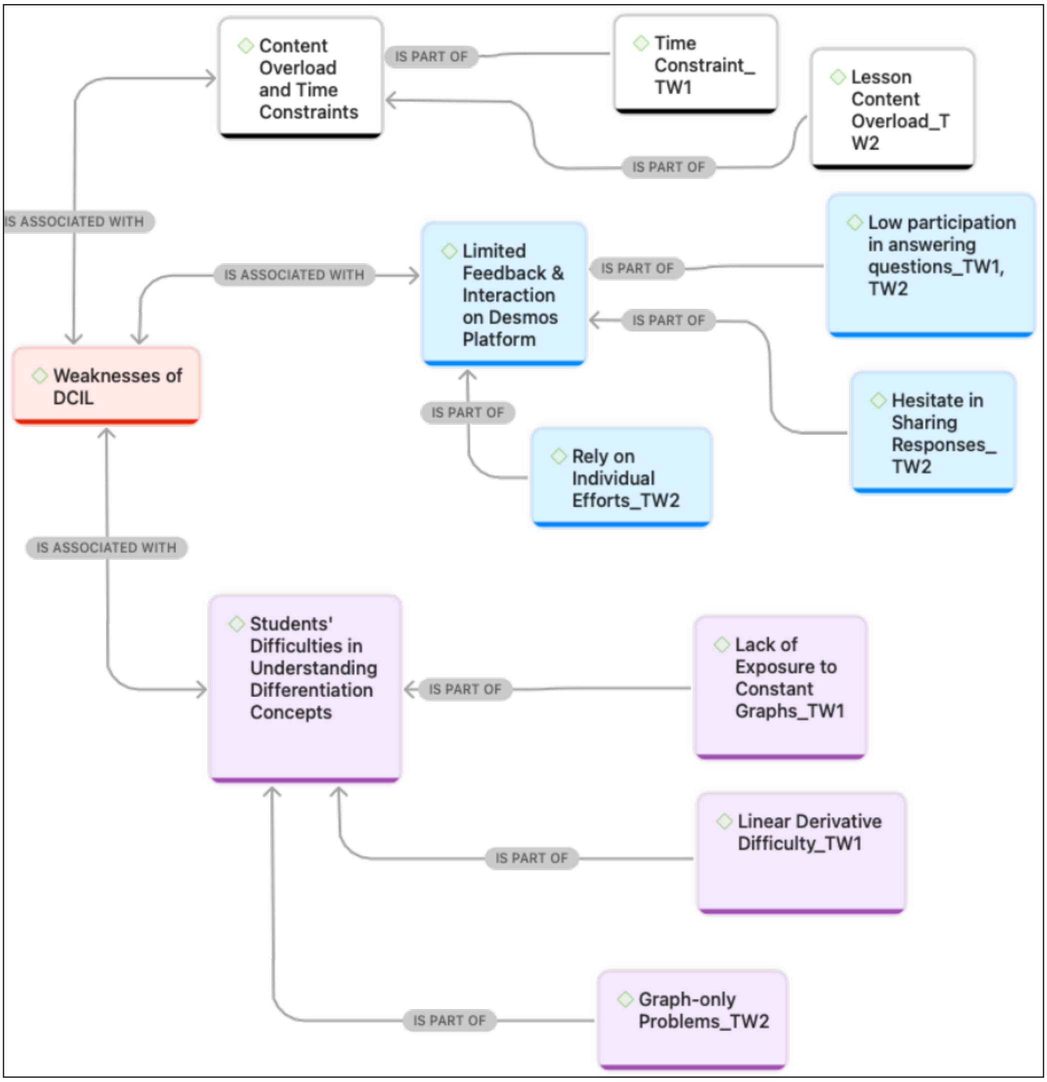

Figure 2. The weaknesses of the DCIL

to understand all of it in just one lesson (Interview L1_TW1: 16).

Teaching Witness 1: Um... the content is too much for them within the two-hour lessons (Interview L1_TW1: 18).

Teaching Witness 2 shared similar concerns about the need for students to have more time to complete the assigned task. She noted that part of the reason for this might be that the lesson contained too much information, particularly in terms of wordiness, making it challenging for students to comprehend.

Teaching Witness 2: ...They seem to understand your instructions, but *they need more time to solve them.* (Interview L3_TW2: 15)

Teaching Witness 2: But there's too much information, too many words.

Researcher: Too much information?

Teaching Witness 2: Yes, it needs to be summarised (Interview L4_TW2: 20-22).

Several difficulties were identified related to students' understanding of the differentiation topic. In Lesson 1, Teaching Witness 1 observed that students encounter difficulties matching original and derivative graphs, particularly when dealing with linear graphs and their derivatives. The constant nature of linear derivatives was identified as a specific point of challenge.

Teaching Witness 1: It's (refer to the derivative of a linear graph) a constant. So, the graph will be in horizontal form.

Researcher: Yes.

Teaching Witness 1: So, for that part of the session, they have difficulty there.

Researcher: Oh… Okay.

Teaching Witness 1: They are a bit confused about this part (Interview L1_TW1: 28-32).

Teaching Witness 2 also discovered that students experienced initial shock when faced with graph-only problems, leading to confusion regarding how to approach tasks.

Teaching Witness 2: Yes, exactly. Because they seem confused at first when they see the graph only; they seem lost, not knowing what to do first (Interview L2_TW2: 25)

Another contributing factor was the lack of exposure to constant graphs in previous classes and lecture notes. This issue led to confusion among students, resulting in challenges when matching linear graphs with their horizontal derivatives.

Teaching Witness 1: Because we seldom expose them to the constant graph. We rarely presented the horizontal derivative graphs to the students (Interview L1_TW1: 58).

Regarding the third sub-theme of the weaknesses of this designed DCIL, which is referred to as limited feedback and interaction on the Desmos platform, both teaching witnesses emphasised the low participation from students, especially in voluntarily answering questions. They provided feedback indicating that only a few students actively participated in answering questions while others remained quiet, leading to uncertainty about their level of engagement.

Teaching Witness 1: Just now, I found out that some of the students were very quiet (Interview L2_TW1: 24).

Teaching Witness 1: Only a few of them answered the questions.

Researcher: Um um…

Teaching Witness 1: For the rest, we were not sure whether they were into the activities or if they understood, but they just preferred to keep quiet (Interview L2_TW1: 26-28).

Teaching Witness 2 pointed out that most groups tended to rely on individual efforts, neglecting the collaborative nature of the tasks. The introverted students exhibited reluctance to actively participate in discussions.

Teaching Witness 2: Yes, introverted students seem less inclined to engage in group discussions. While the platform suits introverted students, it also poses a disadvantage as they may prefer solving problems individually rather than discussing solutions with their peers (Interview OVERALL_TW2: 12)

Teacher Witness 2 also noticed that some students hesitated in submitting their answers, indicating a potential challenge in active participation.

Teaching Witness 2: ... Some students seem afraid to submit their answers immediately. They were just quiet at first (Interview L1_TW2: 17).

Suggested Improvements for DCIL Lesson

From the interview, the teaching witnesses provided feedback on the lessons and suggested improvements for better interaction, aiming to enhance the designed Differential Calculus Instructional Lessons (DCIL). After conducting theme analysis, two sub-themes emerged: Students' Engagement and Interaction Enhancement and Enhancement on Desmos for Better Teaching Practice. Figure 3 shows the connection between the emerged codes and categories.

Teaching Witness 2 recommended that only the group leader should have access to submit answers in group activities to encourage other group members to focus on discussion, thereby increasing students' active participation.

Teaching Witness 2: Okay, for this lesson, regarding the group activity, I think it's better if only the leader of a group can access and submit the answers because some groups seemed uncomfortable with the discussion.

Teaching Witness 2: Exactly. The group members just need to give the answers or share their ideas (Interview L2_TW2: 13&17)

Teaching Witness 1 emphasised that student participation is influenced by attitudes, suggesting the creation of smaller group interactions and problem-solving discussions to foster more active engagement.

Teaching Witness 1: The maximum is 20; having 15 to 20 students in one class is good enough.

Researcher: Oh, okay.

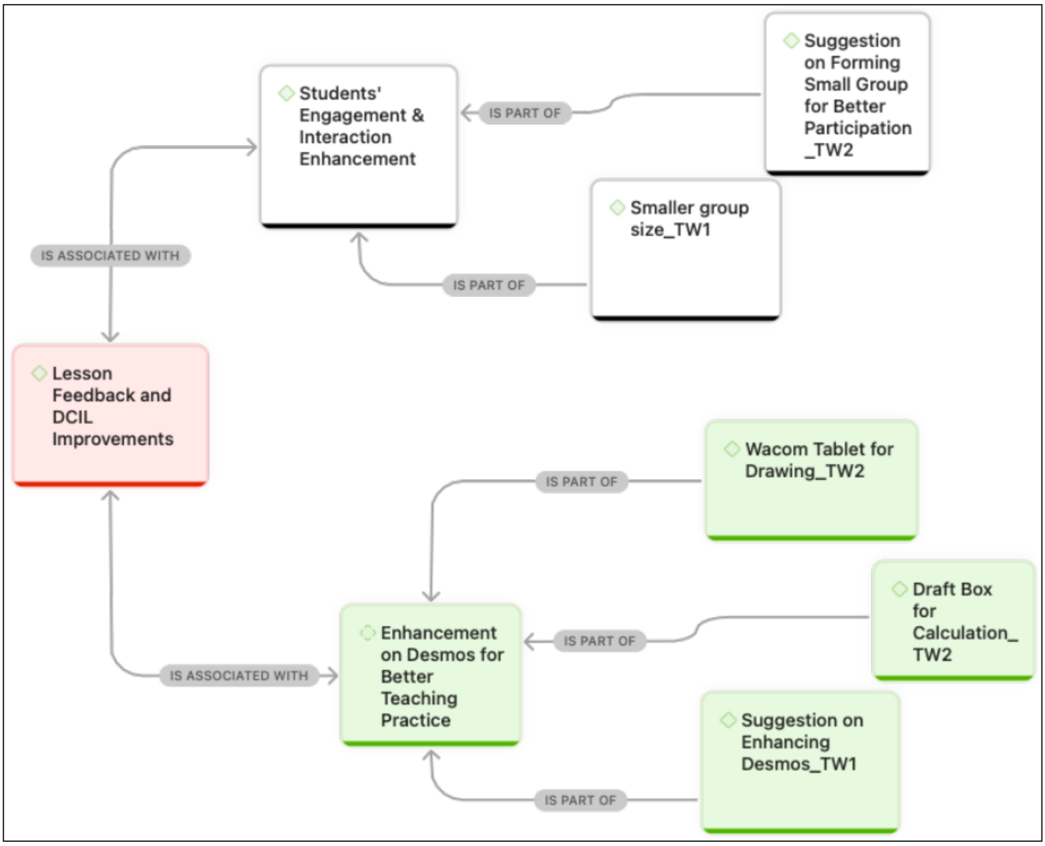

Figure 3. The teaching witnesses' feedback on DCIL

> *Teaching Witness 1: It's also easier for you to monitor their progress* (Interview Overall_TW1: 132-136).

Additionally, Teaching Witness 1 recommended further enhancements to Desmos to improve teaching practices. She proposed integrating a blank whiteboard or digital paper feature to facilitate side-by-side teaching and calculations.

> *Teaching Witness 1: Erm… I'm not sure whether it can be done or not. Maybe there are other features that we can add to Desmos, such as a blank whiteboard or blank paper.*
>
> *Researcher: Okay…*
>
> *Teaching Witness 1: We can arrange it in a side-by-side view, with what you are currently teaching on one side and another view for the whiteboard or the blank screen* (Interview Overall_TW1: 84-88).

On the other hand, Teaching Witness 2 suggested providing a draft box on Desmos for students to calculate and work on their answers before submission, aiming to reduce hesitancy and improve the quality of responses. She also recommended using a Wacom tablet instead of a mouse to draw on the Desmos platform and enhance student interest.

> *Teaching Witness 2: It would be better to have a draft box for students to calculate and work on their answers before submitting them.*
>
> *Some students seem afraid to submit their answers immediately. They were just quiet at first* (Interview L1_TW2: 15&17).
>
> *Teaching Witness 2: Also, I suggest using a Wacom tablet for drawing.*
>
> *Researcher: Ah, okay.*
>
> *Teaching Witness 2: Because students seem more interested when using a tablet rather than a mouse or whiteboard* (Interview L3_TW2: 31-33).

DISCUSSION

The insights derived from interview sessions with two teaching witnesses provided valuable perspectives on the strengths and weaknesses and suggested improvements in the designed Differential Calculus Instructional Lessons (DCIL) to enhance student understanding of derivative concepts and their applications to curve properties. Addressing these identified weaknesses through targeted instruction and modifications to the instructional lessons is expected to contribute to a more accurate and comprehensive understanding of derivative concepts among students.

This outcome was significant, as the previous research review (Bukhatwa et al., 2022; Wang & Hannafin, 2005) has shown that most studies primarily focused on the impact or effectiveness of implementing mathematics-related technologies in schools. This study, however, considered other factors that could contribute to the enhancement of instructional lessons or materials. Thus, it was crucial to incorporate feedback from course lecturers in this study to achieve the desired learning goals.

Findings revealed that the instructional lessons discussed in the papers exhibited several strengths in terms of lesson effectiveness and achievement, interactive teaching strategies, and effective technology use. Teaching Witness 2 emphasised the importance of responsive interactions between students and instructors, emphasising active engagement, feedback provision, and addressing individual needs. This effective instructional feature aligned with the findings of Rolf and Slocum (2021), highlighting that interactive engagement between instructors and students was a key element. Moreover, the study emphasised that effective technology integration enhanced instruction provided alternative assessment methods, and facilitated teacher productivity. Gillani and the collaborators (2008) stated that instructional lessons incorporating technology made instruction more effective, understandable, and meaningful.

According to teaching witnesses, students showed a deficient concept image of the graphical concept, particularly when matching a linear function with its derivative. Although they could compute the derivative of a linear function correctly, they often struggled to recognise and correctly map the horizontal line as the derivative of the linear function. These underdeveloped concept images of the graph shapes led to their conceptual errors (Makonye & Luneta, 2014; Tall & Vinner, 1981).

The instructional observations provided by teaching witnesses highlighted certain weaknesses, such as content overload, time constraints, and limited feedback and interaction on the Desmos platform. Consequently, recommendations were made to enhance instructional design, emphasising student engagement and interaction on Desmos for improved teaching practices.

To address content overload and time constraints in the teaching and learning of mathematics, Keiser and Lambdin (1996) proposed the integration of cooperative learning and problem-solving activities to foster student interaction and engagement. Teaching witnesses in this study also suggested flexible group work presentations, allowing selected groups to present or designating the group leader for answer submission while others focused on group discussions. These recommendations aligned with the importance of flexibility in class scheduling, particularly when implementing innovative teaching methods, such as group work and alternative forms of assessment (Soluk et al., 2022).

Furthermore, strategies to enhance engagement and interaction in the calculus classroom were explored. Cablas (2023) advocated for interactive teaching practices, including purposeful questioning and feedback, known to facilitate interactive learning and increase teacher facilitation and student engagement. The research highlighted that the incorporation of technology in learning environments contributed to increased student engagement, as suggested by Shé, Bhaird, and Fhloinn (2023). Therefore, instructors were urged to fully utilise the Desmos platform, which allowed for visual representations of functions and equations, personalised learning through the teacher dashboard, and immediate feedback (Chorney, 2021; Gulli, 2021). Additionally, sharing anonymous student answers in the classroom through the teacher dashboard in the Desmos platform could enhance engagement and interaction, fostering active participation in problem-solving functions and calculus (Chechan et al., 2023; Liang, 2016). Integration of these strategies enabled educators to create a dynamic and interactive learning environment that enhanced students' engagement and interaction in the calculus classroom.

CONCLUSION

This study adds to the existing literature by offering detailed analyses personalised to design-based research. This study highlighted the importance of teacher witnesses' active participation and contribution to design research to enhance teaching practices and students' learning experiences. The analysis of teaching witnesses during the interview sessions yielded valuable insights for real-time feedback into instructional design and future improvements in the designed instructional lessons for teaching derivative concepts. It highlighted the strengths and weaknesses of the instructional design and offered suggestions for improvement. Consequently, the findings suggest that integrating teaching witnesses can lead to more responsive and effective educational practices, thereby enriching the theoretical implications of DBR in mathematics education.

The limitation of this study is that it relied solely on qualitative interview data to explore the strengths and weaknesses of the designed DCIL and on the analysis of feedback forms provided by the two teaching witnesses to quantify and validate the intervention. This narrow scope restricts the study's conclusions to the specific context in which it was conducted or similar contexts with comparable syllabi. Consequently, the outcomes may not be generalisable to other contexts. However, it does provide a detailed and insightful description derived from the study's context. A recommendation for future study is to include the exploration of students' feedback, as design-based research centres around studying student learning, with their feedback playing a crucial role in instructional design. Furthermore, a longitudinal study involving an additional cycle of teaching experiments in design-based research (DBR) could be conducted to investigate the long-term effects of instructional changes. Additionally, incorporating quantitative methods, such as statistical analyses of student performance in later cycles of DBR, could effectively assess the impact of the designed DCIL on students' outcomes in differential calculus.

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REFERENCES

Arango, J., Gaviria, D., & Valencia, A. (2015). Differential calculus teaching through virtual learning objects in the field of management sciences. *Procedia - Social and Behavioral Sciences*, *176*, 412-418. https://doi.org/10.1016/j.sbspro.2015.01.490

Aspinwall, L., Shaw, K. L., & Presmeg, N. C. (1997). Uncontrollable mental imagery: Graphical connections between a function and its derivative. *Educational Studies in Mathematics*, *33*(3), 301-317. https://doi.org/10.1023/a:1002976729261

Bakker, A., & van Eerde, D. (2015). Chapter 16: An introduction to design-based research with an example from statistics education. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 429-466). Springer. https://doi.org/10.1007/978-94-017-9181-6_16

Bakri, S. R. A., Liew, C. Y., Chen, C. K., Tuh, M. H., & Ling, S. C. (2020). Bridging the gap between the derivatives and graph sketching in Calculus: An innovative game-based learning approach. *Asian Journal of University Education*, *16*(4), 121-136. https://doi.org/10.24191/ajue.v16i4.11962

Bedada, T. B., & Machaba, M. F. (2022). Investigation of student's perception learning calculus with GeoGebra and cycle model. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(10), Article em2164. https://doi.org/10.29333/ejmste/12443

Bibi, A., Ahmad, M., Shahid, W., Zamri, S. N. S., & Abedalaziz, N. A. M. (2019). An evolving research to tackle teaching and learning challenges during differential equations course: A combination of non-routine problems and teacher training. *International Electronic Journal of Mathematics Education*, *14*(3), 647-656. https://doi.org/10.29333/iejme/5777

Block, B.-M., & Mercorelli, P. (2015). Conceptual understanding of complex components and Nyquist-Shannon sampling theorem: A design based research in Engineering. In *2015 IEEE Global Engineering Education Conference* (pp. 462-470). IEEE. https://doi.org/10.1109/EDUCON.2015.7096011

Bogdan, R., & Biklen, S. K. (2007). *Qualitative research for education: An introduction to theory and methods.* Pearson A & B.

Borji, V., Font, V., Alamolhodaei, H., & Sánchez, A. (2018). Application of the complementarities of two theories, APOS and OSA, for the analysis of the university students ' understanding on the graph of the function and its derivative. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(6), 2301-2315. https://doi.org/10.29333/ejmste/89514

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77-101. https://doi.org/10.1191/1478088706qp063oa

Brijlall, D., & Ndlazi, N. J. (2019). Analysing engineering students' understanding of integration to propose a genetic decomposition. *Journal of Mathematical Behavior*, *55*, Article 100690. https://doi.org/10.1016/j.jmathb.2019.01.006

Bukhatwa, B., Al Ruqeishi, E. N. A., & Al Khamisi, F. M. H. (2022). The usefulness of technology-based interactive methods in teaching mathematics and statistics at the college level. *Shanlax International Journal of Education*, *10*(3), 30-40. https://doi.org/10.34293/education.v10i3.4627

Bungum, B., & Sanne, A. (2021). Conditions for the active involvement of teachers in a design-based research project. *Designs for Learning*, *13*(1), 44-54. https://doi.org/10.16993/dfl.169

Cablas, J. C. (2023). The flipped classroom: Enhancing students' learning in teaching calculus. *International Journal of Multidisciplinary: Applied Business and Education Research*, *4*(6), 2014-2022. https://doi.org/10.11594/ijmaber.04.06.26

Chechan, B., Ampadu, E., & Pears, A. (2023). Effect of using Desmos on high school students' understanding and learning of functions. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(10), Article em2331. https://doi.org/10.29333/ejmste/13540

Chorney, S. (2021). Classroom practice and craft knowledge in teaching mathematics using Desmos: challenges and strategies. *International Journal of Mathematical Education in Science and Technology, 53*(12), 3203-3227. https://doi.org/10.1080/0020739X.2021.1931974

Cobb, P., Confrey, J., Disessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, *32*(1), 9-13. https://doi.org/10.3102/0013189X032001009

Fatimah, S., & Yerizon. (2019). Analysis of difficulty learning calculus subject for mathematical education students. *International Journal of Scientific and Technology Research*, *8*(3), 80-84.

Fitriani, Umar, K., Pasaribu, F., Novitasari, W., Samosir, B. S., & Yusmiati. (2023). Analysis of difficulty understanding student Mathematica by using online learning model. *KnE Social Sciences*, 180-187. https://doi.org/10.18502/kss.v8i4.12898

Gillani, S. N., Gujjar, A. A., & Choudhry, B. N. (2008). The use of instructional design in educational technology for effective teaching and learning. *Journal on School Educational Technology*, *3*(3), 49-56. https://doi.org/10.26634/jsch.3.3.736

Goff, W. M., & Getenet, S. (2017). Design-based research in doctoral studies: Adding a new dimension to doctoral research. *International*

Journal of Doctoral Studies, *12*, 107-121. https://doi.org/10.28945/3761

Gulli, C. (2021). Technology in teaching mathematics: Desmos. *Proceedings of GREAT Day*, *2020*(1), Article 8. https://knightscholar.geneseo.edu/proceedings-of-great-day/vol2020/iss1/8

Haghjoo, S., & Reyhani, E. (2021). Undergraduate basic sciences and engineering students' understanding of the concept of derivative. *Journal of Research and Advances in Mathematics Education*, *6*(4), 277-298. https://doi.org/10.23917/jramathedu.v6i4.14093

Hamda, H., Arwadi, F., Sukarna, S., & Syam, R. (2020). Instructional design of multivariable calculus course using Toulmin argument model to increase the understanding of students. *ICLIQE 2020: Proceedings of the 4th International Conference on Learning Innovation and Quality Education*, Article 47. https://doi.org/10.1145/3452144.3453751

Hamid, H. A., Tapsir, R., Shah, R. L. Z. R. M., Salleh, F. Z., Mustaffa, N., Lakim, K. H., & Sayed Abdulhamid, S. R. (2021). Pre-university students' conceptual understanding of functions and derivatives through graphical reasoning. *Review of International Geographical Education Online*, *11*(4), 1013-1022.

Haryani, F., & Hamidah, A. S. (2022). Exploring the impact of technology-integrated mathematics worksheet in the teaching and learning during Covid-19 pandemic. *Mathematics Teaching Research Journal*, *14*(3), 39-59. https://files.eric.ed.gov/fulltext/EJ1361559.pdf

Hashemi, N., Kashefi, H., & Abu, M. S. (2020). The emphasis on generalization strategies in teaching integral: Calculus lesson plans. *Sains Humanika*, *12*(3), 35-43. https://doi.org/10.11113/sh.v12n3.1687

Hitt, F., & Dufour, S. (2021). Introduction to calculus through an open-ended task in the context of speed: representations and actions by students in action. *ZDM - Mathematics Education, 53*(3), 635-647. https://doi.org/10.1007/s11858-021-01258-x

Jamaludin, A. (2016). *Modul motivasi diri* [Self-motivation module]. Dewan Bahasa dan Pustaka.

Jones, S. R., & Watson, K. L. (2018). Recommendations for a "target understanding" of the derivative concept for first-semester calculus teaching and learning. *International Journal of Research in Undergraduate Mathematics Education*, *4*, 199-227. https://doi.org/10.1007/s40753-017-0057-2

Keene, K. A., Hall, W., & Duca, A. (2014). Sequence limits in calculus: Using design research and building on intuition to support instruction. *ZDM Mathematics Education*, *46*(4), 561-574. https://doi.org/10.1007/s11858-014-0597-8

Keiser, J. M., & Lambdin, D. V. (1996). The clock is ticking: Time constraint issues in mathematics teaching reform. *The Journal of Educational Research*, *90*(1), 23-31. https://doi.org/10.1080/00220671.1996.9944440

Kennedy-Clark, S. (2013). Research by design: Design-based research and the higher degree research student. *Journal of Learning Design*, *6*(2), 26-32. https://doi.org/10.5204/jld.v6i2.128

Koller, V., Harvey, S., & Magnotta, M. (2008). *Technology-based learning strategies.* Social Policy Research Associates.

Kwon, O. N., Bae, Y., & Oh, K. H. (2015). Design research on inquiry-based multivariable calculus: Focusing on students' argumentation and instructional design. *ZDM Mathematics Education*, *47*(6), 997-1011. https://doi.org/10.1007/s11858-015-0726-z

Liang, S. (2016). Teaching the concept of limit by using conceptual conflict strategy and Desmos graphing calculator. *International Journal of Research in Education and Science*, *2*(1), 35-48. https://doi.org/10.21890/ijres.62743

Lu, J. C. C. (2023). Using YouTube as an effective educational tool to improve engineering mathematics teaching during the COVID-19 pandemic. *Engineering Proceedings*, *38*(1), Article 24. https://doi.org/10.3390/engproc2023038024

Makgakga, S., & Makwakwa, E. G. (2016). Exploring learners' difficulties in solving grade 12 differential calculus: A case study of one secondary school in Polokwane district. In *ISTE International Conference on Mathematics, Science and Technology Education* (pp. 13-25). Unisa Press. http://hdl.handle.net/10500/22899

Makonye, J. P., & Luneta, K. (2014). Mathematical errors in differential calculus tasks in the Senior School Certificate Examinations in South Africa. *Education as Change*, *18*(1), 119-136. https://doi.org/10.1080/16823206.2013.847014

Malaysian Qualifications Agency. (2019). *Guidelines of Curriculum: Foundation (GCF)*. https://www2.mqa.gov.my/qad/GGP/2019/MQA%20GCF%20FINAL%2027.5.2019%20edited%20(1)_Clean.PDF

Mariano, D. L. F., Barsana, J. G., Dimaculangan, K. A. C., Flores, L. A., Parto, F., Mariano, O. G., & Elipane, L. (2021). Lesson study: A tool for an improved instructional design in teaching integration by parts. *Turkish Journal of Computer and Mathematics Education*, *12*(3), 3862-3869.

Mcgee, D. L., & Moore-Russo, D. (2015). Impact of explicit presentation of slopes in three dimensions on students' understanding of derivatives in multivariable calculus. *International Journal of Science and Mathematics Education*, *13*, 357-384. https://doi.org/10.1007/s10763-014

McKim, C. (2023). Meaningful member-checking: A structured approach to member-checking. *American Journal of Qualitative Research*, *7*(2), 41-52.

Meiliasari, M., Wijayanti, D. A., & Azima, L. A. (2021). An error analysis of students' difficulties in differential calculus. *Journal of Medives : Journal of Mathematics Education IKIP Veteran Semarang*, *5*(1), 48-60. https://doi.org/10.31331/medivesveteran.v5i1.1433

Melnick, S. A., & Meister, D. G. (2008). A comparison of beginning and experienced teachers' concerns. *Educational Research Quarterly, 31*(3), 40-56.

Molina, M., Castro, E., & Castro, E. (2007). Teaching experiments within design research. *The International Journal of Interdisciplinary Social Sciences: Annual Review*, *2*(4), 435-440. https://doi.org/10.18848/1833-1882/CGP/v02i04/52362

Norton, A. H., & McCloskey, A. (2008). Teaching experiments and professional development. *Journal of Mathematics Teacher Education*, *11*(4), 285-305. https://doi.org/10.1007/s10857-008-9076-x

Ojo, A., & Olanipekun, P. (2023). Examining students' concept images in mathematics: The case of undergraduate calculus. *Voice of the Publisher, 9*(4), 242-256. https://doi.org/10.4236/vp.2023.94019

Othman, Z. S. B., Khalid, A. K., & Mahat, A. B. (2018). Students' common mistakes in basic differentiation topics. *AIP Conference Proceedings*, *1974*(1), Article 050009. https://doi.org/10.1063/1.5041709

Pramesti, N., & Dewi, L. (2023). The implementation of understanding by design approach in mathematics learning on elementary school. *Journal of Innovative Mathematics Learning*, *6*(2), 124-131. https://doi.org/10.22460/jiml.v6i2.16304

Rashidi, M. N., Begum, R. A., Mokhtar, M., & Pereira, J. J. (2014). The conduct of structured interviews as research implementation method. *Journal of Advanced Research Design*, *1*(1), 28-34.

Rolf, K. R., & Slocum, T. A. (2021). Features of direct instruction: Interactive lessons. *Behavior Analysis in Practice*, *14*, 793-801. https://doi.org/10.1007/s40617-021-00613-4

Salinas, P., Quintero, E., & Fernández-Cárdenas, J. M. (2016). Fostering dialogue in the calculus classroom using dynamic digital technology. *Digital Experiences in Mathematics Education*, *2*(1), 21-49. https://doi.org/10.1007/s40751-016-0013-9

Setiawan, A. A. (2022). Instructional design: Teaching algebraic equations to grade 8 students with involvement of mathematical reasoning in Cambridge IGCSE curriculum. *Journal of Instructional Mathematics*, *3*(1), 1-15. https://doi.org/10.37640/jim.v3i1.1035

Shahid, I., Beek, A. Van, & McCahan, S. (2022). Observing instructor behaviour in an active learning classroom: A case study of an undergraduate calculus course. *Proceedings 2022 Canadian Engineering Education Association (CEEA-ACEG22) Conference*. Paper 68.

Shé, C. N., Bhaird, C. M. an, & Fhloinn, E. (2023). Factors that influence student engagement with technology-enhanced resources for formative assessments in first-year undergraduate mathematics. *International Journal of Mathematical Education in Science and Technology, 55*(10), 2670-2688. https://doi.org/10.1080/0020739X.2023.2182725

Shé, C. N., Fhloinn, E., & Bhaird, C. M. an. (2023). Student engagement with technology-enhanced resources in mathematics in higher education: A review. *Mathematics*, *11*(3), Article 787. https://doi.org/10.3390/math11030787

Smith, T. W., & Strahan, D. (2004). Toward a prototype of expertise in teaching : A descriptive case study. *Journal of Teacher Education*, *55*(4), 357-371. https://doi.org/10.1177/0022487104267587

Soluk, P., Greenwood-Lee, J., Peschke, J., Beltaos, A., Kumar, V., Muniykwa, K., Babiuk, S., & Zenteno, S. (2022). Building educational resilience in mathematics delivery and assessment (Working paper). In *Pan-Commonwealth Forum 10 (PCF10), 2022*. Commonwealth of Learning. https://doi.org/10.56059/pcf10.616

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology underlying principles and essential elements. In A. E. Kelly & R. A. Lesh (Eds.), *Research design in mathematics and science education* (pp. 267-307). Routledge. https://doi.org/10.4324/9781410602725

Stephan, M. L. (2015). Conducting classroom design research with teachers. *ZDM - Mathematics Education*, *47*, 905-917. https://doi.org/10.1007/s11858-014-0651-6

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151-169. https://doi.org/10.1007/BF00305619

The Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, *32*(1), 5-8. https://doi.org/10.3102/0013189X032001005

Thomas, D. R. (2017). Feedback from research participants: Are member checks useful in qualitative research? *Qualitative Research in Psychology*, *14*(1), 23-41. https://doi.org/10.1080/14780887.2016.1219435

Wagner, E. R., Orme, S. M., Turner, H. J., & Yopp, D. (2017). Encouraging example generation: A teaching experiment in first-semester calculus. *Problems, Resources and Issues in Mathematics Undergraduate Studies*, *27*(2), 212-234. https://doi.org/10.1080/10511970.2016.1194340

Wang, F., & Hannafin, M. J. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology Research and Development*, *53*(4), 5-23. https://doi.org/10.1007/BF02504682